Headway statistics of public transport in Mexican cities

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Abstract

We present a cellular automaton simulating the behavior of public bus transport in several Mexican cities. The headway statistics obtained from the model is compared to the measured time intervals between subsequent bus arrivals to a given bus stop and to a spacing distribution resulting from a random matrix theory.

KEYWORDS: transport, cellular automata, random matrix theory

The public transport in Mexico is organized differently to that known in Europe. First, no leading companies are responsible for the city transport. Thus, there are no timetables for the city buses and sometimes even not well defined bus stops. Moreover, the driver is usually the owner of the bus and so his aim is to maximize the income. Since every passenger entering the bus has to pay, the driver tries to collect the largest possible number of passengers. When not regulated the time interval under which two subsequent buses pass a given point will display a Poissonian distribution. This is a consequence of the absence of correlations between the motion of different buses. Such situation is, however, not welcomed by the drivers since then the probability density that two buses arrive to a bus stop within short time interval is large. In this case the first bus collects all waiting passengers and the second one that arrives bit later will find the stop practically empty. This simple reasoning makes clear that existence of certain correlations between buses, that will change the Poisson process, will be of favor. Indeed, in Mexico various strategies have been developed to create such correlations. Here we discuss the situation in three cities: Cuernavaca, Puebla and Mexico City.

In Puebla there is not an additive mechanism that helps to increase the bus correlation. Hence the headway statistics of the buses in this city should be close to Poissonian. In Cuernavaca in turn, the information about the times, when the buses pass certain points, are notified and then sold to the bus drivers (there is a real market with this information). The driver can in such a way change the velocity of the bus in dependence on the position of the bus in front of him and a bus-bus correlation appears.

To describe the correlations we use a modified version of the celebrated Nagel-Schreckenberg cellular automaton (see [1] and [2]). Consider N equal cells on the line (for our purpose we define the length of one cell as 30 meters) and n particles (buses) moving along it. Thus, $c = \frac{n}{N}$ is the bus density and $\bar{d} = c^{-1}$ is the mean distance between two neighboring buses. Furthermore, define the maximal velocity v_{max} of the bus and its probability p to slow down $p \in [0, 1]$. The Nagel-Schreckenberg model describes the dynamics of the system with the help of the following update rules. In time $t = 0$ the positions x_i of the particle $i = 1 \dots n$ are integer numbers randomly chosen from the set $1 \dots N$ satisfying the condition $x_{i-1} > x_i$ for every $i = 2 \dots n$. Furthermore, in time $t = 0$ the velocity v_i of the i-th particle is set to zero for all $i : v_i(0) = 0$. The buses start to move with velocity v according to the update rules that has to be applied simultaneously to all particles.

- Step one: When velocity of the bus is smaller than a maximal velocity v_{max}, it increases its velocity by one

$$v_i(t+1) = \min \{v_i(t) + 1; v_{max}\} \tag{1}$$

- Step two: Particles with positive velocities are randomly slowed down

$$v_i(t+1) = v_i(t+1) - 1 \tag{2}$$

  with probability p

- Step three: Particles update their positions according to

$$x_i(t+1) = \min \{x_i(t) + v_i(t+1); x_{i-1}(t+1) - 1\}, \tag{3}$$

  i.e., the particles move according to the rule $x_i(t+1) = x_i(t) + v_i(t+1)$ with the restriction that they cannot occupy the same cell or overtake each other. In that case the particle hops to the cell behind the occupied one.

To create a modification of the model and to adapt it to the situation in Mexico we change the update rules on a subset M of possible bus positions where the information is passed to the driver. (The density of those points will be denoted by a; $a = M/N$.)

To change the model we add to the steps (1),(2),(3) an additive step (4) that takes into account the processing of the information :

At the points $j \in M$ the information about the time interval Δt to a preceding bus, which passed that point, is available to the driver. Using it the driver can modify the bus velocity - to speed up if $\Delta t > \bar{t}$ or to slows down for $\Delta t < \bar{t}$. (Recall that $\bar{t}$ is the mean time interval between subsequent buses.) Hence we change the model by adding

- Step four:

$$v_j(t) = v_j(t) + 1$$

  if $t_i(j) - t_{i-1}(j) < \bar{t}$

$$v_j(t) = v_j(t) - 1 \tag{4}$$

  if $t_i(j) - t_{i-1}(j) > \bar{t}$ where $t_i(j)$ denotes the time when a bus i passed through the point j: $x_i(t_i(j)) = j$.

Using the modified model we focus on the *headway statistics*, i.e. on the probability distribution of the time intervals Δt between two subsequent buses passing a given point and compare it with the results obtained in the cities. For the simulation of the transport in Puebla we choose $c = 1/40$, $v_{max} = 2, p = 0.5$, and $a = 0$ taking into account the fact that there are no checking points. It is not surprising that the headway statistics is in this case close to the Poisson distribution (see Figure 1)

$$P(t) = e^{-t}, \tag{5}$$

when t is the spacing re-scaled to the mean distance equal 1. Besides the headway statistics, we compare also a *number variance* $\Sigma^2(t)$ that is defined as

$$\Sigma^2(t) = \langle (n(t) - t)^2 \rangle, \tag{6}$$

where $n(t)$ is the number of bus arrivals to a given point during the time period of the length t. Note that $\langle n(t) \rangle = t$ due to the fact that $\langle t \rangle = 1$. It can easily be checked that for a Poissonian process

$$\Sigma^2(t) = t. \tag{7}$$

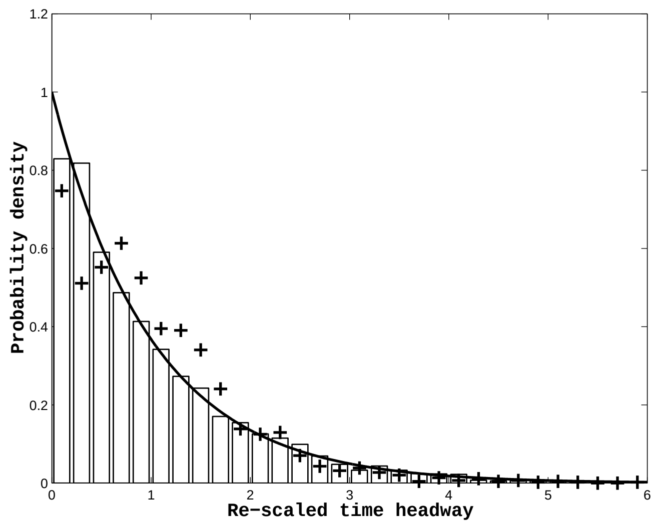

FIG.1. Time headway of the bus transport in Puebla. The curve represents the Poisson distribution (5). Plus signs display the time headway distribution of buses in Puebla and bars are taken from the CA model with $a = 0$.

The number variance obtained from the data and from the simulation fits quite well with this prediction (see Figure 3). The data show, however, a small deviations from (7). This is a manifestation of a weak interaction between the buses which probably originates from the fact that the buses interact through the number of passengers waiting on the stops. Namely when the distance between the buses is large, more passengers are waiting in the stop and the delay of bus in the stopping-place is longer.

To simulate the situation in Cuernavaca and in Mexico City we use parameters $c = 1/40$, $v_{max} = 2, p = 0.5$, $a = 1/36$, which represents one bus per 1.2 kilometers and one check point per one kilometer approximately. The modified cellular automaton leads in this case to significant changes in the time interval distribution (see Figure 2). The distribution obtained from the automaton fits well the observed time interval distribution. Moreover both distributions conform well with the distribution (see Ref. [3])

$$P(t) = \frac{32}{\pi^2} t^2 e^{\frac{4}{\pi} t^2}, \tag{8}$$

that describes the spacing distribution of a Gaussian unitary ensemble of random matrices (GUE). (It is known that this function describes also the distance distribution of certain one-dimensional interacting gases (see Ref. [5]).

Similar agreement is observed also for the number variance (see Figure 3) where GUE leads to

$$\Sigma^2(t) = \frac{1}{\pi^2}(\ln 2\pi t + \gamma + 1) \tag{9}$$

with $\gamma \approx 0.57721566$ (see [4]).

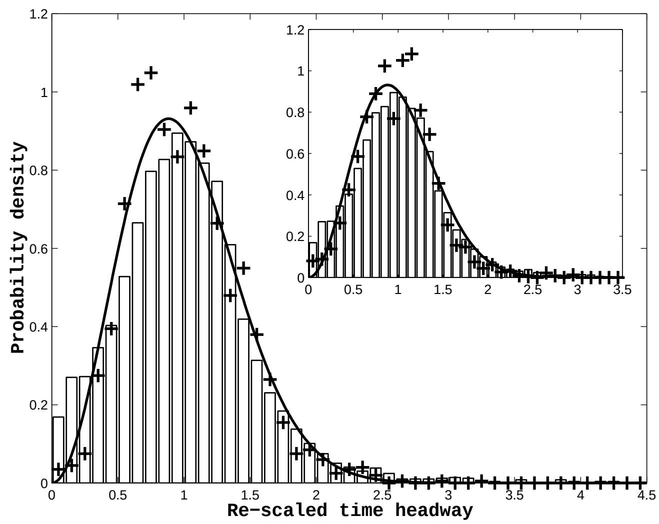

FIG.2. Time headway of bus transport in Cuernavaca and Mexico City.
The curve represents the Wigner formula (8.) Plus signs display the time headway distribution of buses in Cuernavaca and bars represent the results of the CA model with $a = 1/36$. The results obtained for buses in Mexico City are presented on the inset.

However, as evident on the Figure 3, the interaction between the buses in Cuernavaca and in Mexico City is stronger than that resulting from the cellular model and leads to the stronger correlations. Whereas in the automaton model the correlations exist between the nearest neighbors only, in Cuernavaca and Mexico City one can recognize that interaction exists also between the first, second and third neighbor.

We conclude that the modified Nagel-Schreckenberg cellular automaton successfully describes microscopic properties of the bus transport in some Mexican cities. The velocity of the buses is influenced by the information about their mutual positions so that the drivers can optimize their rank in competing on the passengers to be transported. This finally increase the coordination of the bus motion and changes the time headway statistics.

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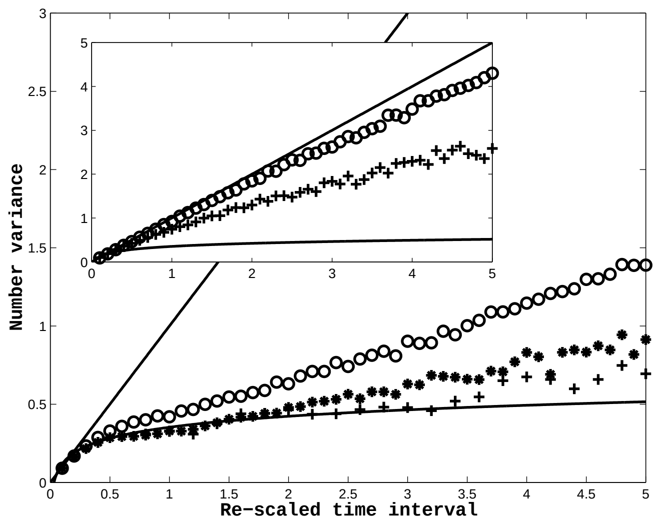

FIG.3. Number variance of the public transport in some Mexican cities.
Curve and line represent the prediction of the RMT (9) and (7), respectively. Plus signs and stars display number variance obtained from the public bus transport in Cuernavaca and Mexico City respectively. Circles are the results of the CA model for $a = 1/36$. The number variance obtained from the transport in Puebla is displayed on the insert (plus signs).The circles show the result of the CA model obtained for $a = 0$.

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